Problems in laser physics

Sheet 8

Handed out on 11. 1. 18 for the Tutorial on 25. 1. 18

Problem 22: Passive Q-switching (4P)

A saturable absorber can be seen as a three-level laser scheme, in which the pump radiation is given by the laser line of the Q-switched laser. Thus the transmission is low in the beginning, and increases with incident photon flux as the excited ions are stored in an upper level with a long lifetime, reducing the ground-state population. For an ideal saturable absorber, the dependence of the final transmission on the incident fluence has been derived as

$$T(J) = \frac{J_{sat}}{J} \ln \left[1 + \left(e^{\frac{J}{J_{sat}}} - 1 \right) T_0 \right] , \tag{1}$$

with T_0 being the initial, i.e. unpumped, transmission of the saturable medium and J_{sat} the saturation fluence. The initial absorption can be derived by the absorber density N^*, the absorption cross section σ_a^* of the saturable absorber and its length L^*, as

$$T_0 = e^{-\sigma_a^* N^* L^*} . \tag{2}$$

However, in most saturable absorber media the final transmission never reaches 100% at high input fluences because an additional absorption is present on the laser line starting from the excited absorber state. This excited-state absorption (ESA) with a cross section $\sigma_{a,ESA}^*$ results in a maximum transmission of

$$T_{max} = e^{-\sigma_{a,ESA}^* N^* L^*} . \tag{3}$$

In this case, the functional dependence of the absorber transmission is given by

$$T_{real} = T_0 + \frac{T(J) - T_0}{1 - T_0} (T_{max} - T_0) . \tag{4}$$

A well known saturable absorber used for passive Q-switching of a Nd:YAG laser at 1.064 μm is Cr^{4+}:YAG, showing $\sigma_a^* = 7 \times 10^{-18}$ cm^2 and

$\sigma^*_{a,ESA} = 2 \times 10^{-18}\ \mathrm{cm}^2$.
(a) Calculate the saturation fluence neglecting stimulated back-emission. (1P)
(b) For a $2.5\ \mathrm{mm}$-thick sample an initial transmission of 60% was measured. Derive the absorber ion density. (1P)
(c) Sketch the transmission of both relations (with and without taking ESA into account) on a linear and logarithmic fluence scale ($J = 0.01 - 1\ \frac{\mathrm{J}}{\mathrm{cm}^2}$) for the case of b). (2P)

Problem 23: Chirped pulses (4P)

A Ti:sapphire laser emits $\tau_p = 15\ \mathrm{fs}$ pulses at a central wavelength of $\lambda_0 = 800\ \mathrm{nm}$. After passing some optical elements, this pulse exhibits a chirp corresponding to $b = 0.01\frac{\omega_0}{\tau_p}$.

(a) Calculate the central laser frequency ω_0, the linear chirp parameter b and the Gaussian parameter ξ (1P).
(b) Calculate the pulse bandwidth in frequency and wavelength and the time-bandwidth product (2P).
(c) Using methods of pulse compression, to which minimum pulse width can this pulse be compressed theoretically (1P)?

Problem 24: Kerr lens self focusing (4P)

In a nonlinear medium, the Kerr effect will create a refractive index difference $\Delta n = n_2 I$, which causes the beam to focus for $n_2 > 0$. We can see this focusing as a total-internal reflection with the critical angle θ_c, while simultaneously a beam confined to the diameter D will show a diffraction with an angle θ_D. Both are given by

$$\cos\theta_c = \frac{1}{1+\frac{\Delta n}{n_0}} \quad \text{and} \quad \theta_D \approx 1.22\frac{\lambda}{n_0 D}\ . \tag{5}$$

When the nonlinear effect dominates over the diffraction, the beam will self focus to a single point, causing catastrophic damage to the medium. (a) Assuming $\frac{\Delta n}{n_0} \ll 1$ and $\theta_c \ll 1$, show that this self-focusing effect occurs for laser powers passing the critical power of (3P)

$$P_{cr} \approx 1.49\pi\frac{\lambda^2}{8 n_0 n_2}\ . \tag{6}$$

(b) Calculate that power for silica glass ($n_0 = 1.48$, $n_2 = 2.1 \times 10^{-16}\ \frac{\mathrm{cm}^2}{\mathrm{W}}$) at $\lambda = 800\ \mathrm{nm}$ (1P).